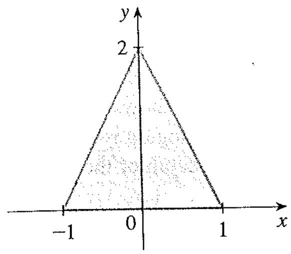
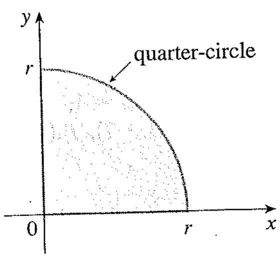
31. $\rho = 1$



32. $\rho = 2$



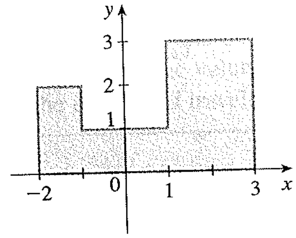
33. Find the centroid of the region bounded by the curves $y = 2^x$ and $y = x^2$, $0 \leqslant x \leqslant 2$, to three decimal places. Sketch the region and plot the centroid to see if your answer is reasonable.

34. Use a graph to find approximate x-coordinates of the points of intersection of the curves $y = x + \ln x$ and $y = x^3 - x$. Then find (approximately) the centroid of the region bounded by these curves.

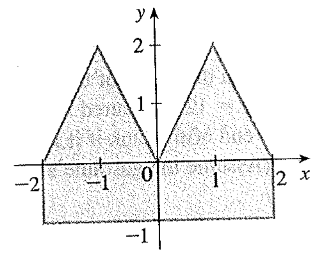
35. Prove that the centroid of any triangle is located at the point of intersection of the medians. [*Hints:* Place the axes so that the vertices are $(a, 0)$, $(0, b)$, and $(c, 0)$. Recall that a median is a line segment from a vertex to the midpoint of the opposite side. Recall also that the medians intersect at a point two-thirds of the way from each vertex (along the median) to the opposite side.]

36–37 □ Find the centroid of the region shown, not by integration, but by locating the centroids of the rectangles and triangles (from Exercise 35) and using additivity of moments.

36.



37.



38–40 □ Use the Theorem of Pappus to find the volume of the given solid.

38. A sphere of radius r (Use Example 4.)

39. A cone with height h and base radius r

40. The solid obtained by rotating the triangle with vertices $(2, 3)$, $(2, 5)$, and $(5, 4)$ about the x-axis

41. Prove Formulas 9.

42. Let $\mathcal{R}$ be the region that lies between the curves $y = x^m$ and $y = x^n$, $0 \leqslant x \leqslant 1$, where m and n are integers with $0 \leqslant n < m$.

(a) Sketch the region $\mathcal{R}$.

(b) Find the coordinates of the centroid of $\mathcal{R}$.

(c) Try to find values of m and n such that the centroid lies *outside* $\mathcal{R}$.

8.4 Applications to Economics and Biology

In this section we consider some applications of integration to economics (consumer surplus) and biology (blood flow, cardiac output). Others are described in the exercises.

Consumer Surplus

Recall from Section 4.8 that the demand function $p(x)$ is the price that a company has to charge in order to sell x units of a commodity. Usually, selling larger quantities requires lowering prices, so the demand function is a decreasing function. The graph of a typical demand function, called a **demand curve**, is shown in Figure 1. If X is the amount of the commodity that is currently available, then $P = p(X)$ is the current selling price.

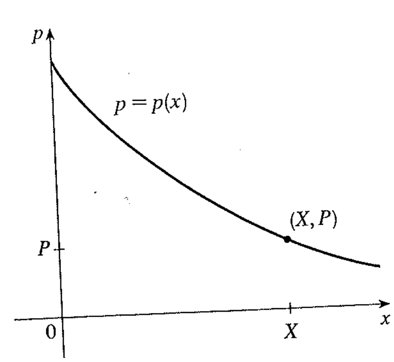


FIGURE 1
A typical demand curve

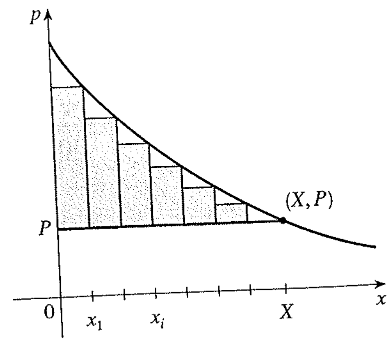


FIGURE 2

We divide the interval $[0, X]$ into n subintervals, each of length $\Delta x = X/n$, and let $x_i^* = x_i$ be the right endpoint of the ith subinterval, as in Figure 2. If, after the first x_{i-1} units were sold, a total of only x_i units had been available and the price per unit had been set at $p(x_i)$ dollars, then the additional Δx units could have been sold (but no more). The consumers who would have paid $p(x_i)$ dollars placed a high value on the product; they would have paid what it was worth to them. So, in paying only P dollars they have saved an amount of

$$(\text{savings per unit})(\text{number of units}) = [p(x_i) - P]\,\Delta x$$

Considering similar groups of willing consumers for each of the subintervals and adding the savings, we get the total savings:

$$\sum_{i=1}^{n} [p(x_i) - P]\,\Delta x$$

(This sum corresponds to the area enclosed by the rectangles in Figure 2.) If we let $n \to \infty$, this Riemann sum approaches the integral

$$\boxed{1} \qquad \int_0^X [p(x) - P]\,dx$$

which economists call the **consumer surplus** for the commodity.

The consumer surplus represents the amount of money saved by consumers in purchasing the commodity at price P, corresponding to an amount demanded of X. Figure 3 shows the interpretation of the consumer surplus as the area under the demand curve and above the line $p = P$.

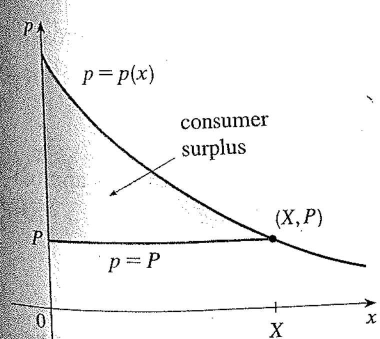


FIGURE 3

EXAMPLE 1 □ The demand for a product, in dollars, is

$$p = 1200 - 0.2x - 0.0001x^2$$

Find the consumer surplus when the sales level is 500.

SOLUTION Since the number of products sold is $X = 500$, the corresponding price is

$$P = 1200 - (0.2)(500) - (0.0001)(500)^2 = 1075$$

Therefore, from Definition 1, the consumer surplus is

$$\begin{aligned}\int_0^{500} [p(x) - P]\,dx &= \int_0^{500} (1200 - 0.2x - 0.0001x^2 - 1075)\,dx \\ &= \int_0^{500} (125 - 0.2x - 0.0001x^2)\,dx \\ &= 125x - 0.1x^2 - (0.0001)\left(\frac{x^3}{3}\right)\Bigg]_0^{500} \\ &= (125)(500) - (0.1)(500)^2 - \frac{(0.0001)(500)^3}{3} \\ &= \$33{,}333.33\end{aligned}$$

Blood Flow

In Example 7 in Section 3.3 we discussed the law of laminar flow:

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

which gives the velocity v of blood that flows along a blood vessel with radius R and length l at a distance r from the central axis, where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood. Now, in order to compute the rate of blood flow, or *flux* (volume per unit time), we consider smaller, equally spaced radii $r_1, r_2, \ldots$. The approximate area of the ring (or washer) with inner radius r_{i-1} and outer radius r_i is

$$2\pi r_i\,\Delta r \qquad \text{where} \quad \Delta r = r_i - r_{i-1}$$

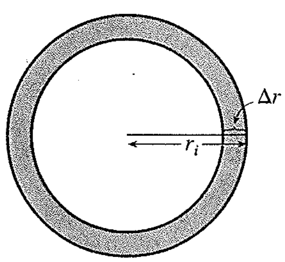


FIGURE 4

(See Figure 4.) If Δr is small, then the velocity is almost constant throughout this ring and can be approximated by $v(r_i)$. Thus, the volume of blood per unit time that flows across the ring is approximately

$$(2\pi r_i\,\Delta r)\,v(r_i) = 2\pi r_i v(r_i)\,\Delta r$$

and the total volume of blood that flows across a cross-section per unit time is approximately

$$\sum_{i=1}^{n} 2\pi r_i v(r_i)\,\Delta r$$

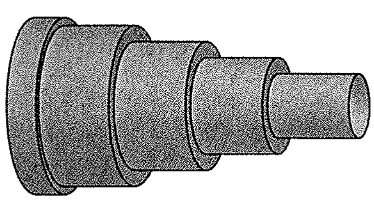


FIGURE 5

This approximation is illustrated in Figure 5. Notice that the velocity (and hence the volume per unit time) increases toward the center of the blood vessel. The approximation gets better as n increases. When we take the limit we get the exact value of the **flux** (or *discharge*), which is the volume of blood that passes a cross-section per unit time:

$$\begin{aligned}F &= \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi r_i v(r_i)\,\Delta r \\ &= \int_0^R 2\pi r v(r)\,dr \\ &= \int_0^R 2\pi r \frac{P}{4\eta l}(R^2 - r^2)\,dr\end{aligned}$$

$$= \frac{\pi P}{2\eta l} \int_0^R (R^2 r - r^3)\, dr = \frac{\pi P}{2\eta l} \left[R^2 \frac{r^2}{2} - \frac{r^4}{4} \right]_{r=0}^{r=R}$$

$$= \frac{\pi P}{2\eta l} \left[\frac{R^4}{2} - \frac{R^4}{4} \right] = \frac{\pi P R^4}{8\eta l}$$

The resulting equation

$$\boxed{2} \qquad F = \frac{\pi P R^4}{8\eta l}$$

is called **Poiseuille's Law**; it shows that the flux is proportional to the fourth power of the radius of the blood vessel.

Cardiac Output

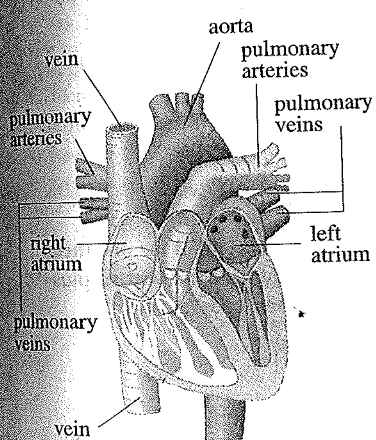


FIGURE 6

Figure 6 shows the human cardiovascular system. Blood returns from the body through the veins, enters the right atrium of the heart, and is pumped to the lungs through the pulmonary arteries for oxygenation. It then flows back into the left atrium through the pulmonary veins and then out to the rest of the body through the aorta. The **cardiac output** of the heart is the volume of blood pumped by the heart per unit time, that is, the rate of flow into the aorta.

The *dye dilution method* is used to measure the cardiac output. Dye is injected into the right atrium and flows through the heart into the aorta. A probe inserted into the aorta measures the concentration of the dye leaving the heart at equally spaced times over a time interval $[0, T]$ until the dye has cleared. Let $c(t)$ be the concentration of the dye at time t. If we divide $[0, T]$ into subintervals of equal length Δt, then the amount of dye that flows past the measuring point during the subinterval from $t = t_{i-1}$ to $t = t_i$ is approximately

$$(\text{concentration})(\text{volume}) = c(t_i)(F\,\Delta t)$$

where F is the rate of flow that we are trying to determine. Thus, the total amount of dye is approximately

$$\sum_{i=1}^{n} c(t_i) F\,\Delta t = F \sum_{i=1}^{n} c(t_i)\,\Delta t$$

and, letting $n \to \infty$, we find that the amount of dye is

$$A = F \int_0^T c(t)\, dt$$

Thus, the cardiac output is given by

$$\boxed{3} \qquad F = \frac{A}{\displaystyle\int_0^T c(t)\, dt}$$

where the amount of dye A is known and the integral can be approximated from the concentration readings.

t	$c(t)$	t	$c(t)$
0	0	6	6.1
1	0.4	7	4.0
2	2.8	8	2.3
3	6.5	9	1.1
4	9.8	10	0
5	8.9		

EXAMPLE 2 □ A 5-mg bolus of dye is injected into a right atrium. The concentration of the dye (in milligrams per liter) is measured in the aorta at one-second intervals as shown in the chart. Estimate the cardiac output.

SOLUTION Here $A = 5$, $\Delta t = 1$, and $T = 10$. We use Simpson's Rule to approximate the integral of the concentration:

$$\int_0^{10} c(t)\,dt \approx \tfrac{1}{3}[0 + 4(0.4) + 2(2.8) + 4(6.5) + 2(9.8) + 4(8.9)$$
$$+ 2(6.1) + 4(4.0) + 2(2.3) + 4(1.1) + 0]$$
$$\approx 41.87$$

Thus, Formula 3 gives the cardiac output to be

$$F = \frac{A}{\int_0^{10} c(t)\,dt} \approx \frac{5}{41.87}$$
$$\approx 0.12 \text{ L/s} = 7.2 \text{ L/min}$$

□

8.4 Exercises

1. The marginal cost function $C'(x)$ was defined to be the derivative of the cost function. (See Sections 3.3 and 4.8.) If the marginal cost of manufacturing x units of a product is $C'(x) = 0.006x^2 - 1.5x + 8$ (measured in dollars per unit) and the fixed start-up cost is $C(0) = \$1{,}500{,}000$, use the Total Change Theorem to find the cost of producing the first 2000 units.

2. The marginal revenue from selling x items is $90 - 0.02x$. The revenue from the sale of the first 100 items is \$8800. What is the revenue from the sale of the first 200 items?

3. The marginal cost of producing x units of a certain product is $74 + 1.1x - 0.002x^2 + 0.00004x^3$ (in dollars per unit). Find the increase in cost if the production level is raised from 1200 units to 1600 units.

4. The demand function for a certain commodity is $p = 5 - x/10$. Find the consumer surplus when the sales level is 30. Illustrate by drawing the demand curve and identifying the consumer surplus as an area.

5. A demand curve is given by $p = 450/(x + 8)$. Find the consumer surplus when the selling price is \$10.

6. The **supply function** $p_S(x)$ for a commodity gives the relation between the selling price and the number of units that manufacturers will produce at that price. For a higher price, manufacturers will produce more units, so p_S is an increasing function of x. Let X be the amount of the commodity currently produced and let $P = p_S(X)$ be the current price. Some producers would be willing to make and sell the commodity for a lower selling price and are therefore receiving more than their minimal price. The excess is called the **producer surplus**. An argument similar to that for consumer surplus shows that the surplus is given by the integral

$$\int_0^X [P - p_S(x)]\,dx$$

Calculate the producer surplus for the supply function $p_S(x) = 3 + 0.01x^2$ at the sales level $X = 10$. Illustrate by drawing the supply curve and identifying the producer surplus as an area.

7. A supply curve is given by $p = 5 + \frac{1}{10}\sqrt{x}$. Find the producer surplus when the selling price is \$10.

8. For a given commodity and pure competition, the number of units produced and the price per unit are determined as the coordinates of the point of intersection of the supply and demand curves. Given the demand curve $p = 50 - x/20$ and the supply curve $p = 20 + x/10$, find the consumer surplus and the producer surplus. Illustrate by sketching the supply and demand curves and identifying the surpluses as areas.

9. A company modeled the demand curve for its product (in dollars) with

$$p = \frac{800{,}000e^{-x/5000}}{x + 20{,}000}$$

Use a graph to estimate the sales level when the selling price is \$16. Then find (approximately) the consumer surplus for this sales level.

10. A movie theater has been charging \$7.50 per person and selling about 400 tickets on a typical weeknight. After surveying

their customers, the theater estimates that for every 50 cents that they lower their price, the number of moviegoers will increase by 35 per night. Find the demand function and calculate the consumer surplus when the tickets are priced at $6.00.

11. If the amount of capital that a company has at time t is $f(t)$, then the derivative, $f'(t)$, is called the *net investment flow.* Suppose that the net investment flow is $\sqrt{t}$ million dollars per year (where t is measured in years). Find the increase in capital (the *capital formation*) from the fourth year to the eighth year.

12. A hot, wet summer is causing a mosquito population explosion in a lake resort area. The number of mosquitos is increasing at an estimated rate of $2200 + 10e^{0.8t}$ per week (where t is measured in weeks). By how much does the mosquito population increase between the fifth and ninth weeks of summer?

13. Use Poiseuille's Law to calculate the rate of flow in a small human artery where we can take $\eta = 0.027$, $R = 0.008$ cm, $l = 2$ cm, and $P = 4000$ dynes/cm^2.

14. High blood pressure results from constriction of the arteries. To maintain a normal flow rate (flux), the heart has to pump harder, thus increasing the blood pressure. Use Poiseuille's Law to show that if R_0 and P_0 are normal values of the radius and pressure in an artery and the constricted values are R and P, then for the flux to remain constant, P and R are related by the equation

$$\frac{P}{P_0} = \left(\frac{R_0}{R}\right)^4$$

Deduce that if the radius of an artery is reduced to three-fourths of its former value, then the pressure is more than tripled.

15. The dye dilution method is used to measure cardiac output with 8 mg of dye. The dye concentrations, in mg/L, are modeled by $c(t) = \frac{1}{4}t(12 - t)$, $0 \leqslant t \leqslant 12$, where t is measured in seconds. Find the cardiac output.

16. After an 8-mg injection of dye, the readings of dye concentration at two-second intervals are as shown in the table. Use Simpson's Rule to estimate the cardiac output.

t	$c(t)$	t	$c(t)$
0	0	12	3.9
2	2.4	14	2.3
4	5.1	16	1.6
6	7.8	18	0.7
8	7.6	20	0
10	5.4		

8.5 Probability

Calculus plays a role in the analysis of random behavior. Suppose we consider the cholesterol level of a person chosen at random from a certain age group, or the height of an adult female chosen at random, or the lifetime of a randomly chosen battery of a certain type. Such quantities are called **continuous random variables** because their values actually range over an interval of real numbers, although they might be measured or recorded only to the nearest integer. We might want to know the probability that a blood cholesterol level is greater than 250, or the probability that the height of an adult female is between 60 and 70 inches, or the probability that the battery we are buying lasts between 100 and 200 hours. If X represents the lifetime of that type of battery, we denote this last probability as follows:

$$P(100 \leqslant X \leqslant 200)$$

According to the frequency interpretation of probability, this number is the long-run proportion of all batteries of the specified type whose lifetimes are between 100 and 200 hours. Since it represents a proportion, the probability naturally falls between 0 and 1.

Every continuous random variable X has a **probability density function** f. This means that the probability that X lies between a and b is found by integrating f from a to b:

$$\boxed{1} \qquad P(a \leqslant X \leqslant b) = \int_a^b f(x)\,dx$$

For example, Figure 1 shows the graph of a model of the probability density function f for a random variable X defined to be the height in inches of an adult female in the United States (according to data from the National Health Survey). The probability that the height

41. Suppose the region lies between two curves $y = f(x)$ and $y = g(x)$ where $f(x) \geq g(x)$, as illustrated in Figure 13. Choose points x_i with $a = x_0 < x_1 < \cdots < x_n = b$ and choose x_i^* to be the midpoint of the ith subinterval; that is, $x_i^* = \overline{x}_i = \frac{1}{2}(x_{i-1} + x_i)$. Then the centroid of the ith approximating rectangle R_i is its center $C_i = \left(\overline{x}_i, \frac{1}{2}[f(\overline{x}_i) + g(\overline{x}_i)]\right)$. Its area is $[f(\overline{x}_i) - g(\overline{x}_i)]\,\Delta x$, so its mass is $\rho\,[f(\overline{x}_i) - g(\overline{x}_i)]\,\Delta x$. Thus, $M_y(R_i) = \rho\,[f(\overline{x}_i) - g(\overline{x}_i)]\,\Delta x \cdot \overline{x}_i = \rho\overline{x}_i\,[f(\overline{x}_i) - g(\overline{x}_i)]\,\Delta x$ and $M_x(R_i) = \rho\,[f(\overline{x}_i) - g(\overline{x}_i)]\,\Delta x \cdot \frac{1}{2}[f(\overline{x}_i) + g(\overline{x}_i)] = \rho \cdot \frac{1}{2}\left[f(\overline{x}_i)^2 - g(\overline{x}_i)^2\right]\Delta x$. Summing over i and taking the limit as $n \to \infty$, we get $M_y = \lim\limits_{n\to\infty} \sum_i \rho\overline{x}_i\,[f(\overline{x}_i) - g(\overline{x}_i)]\,\Delta x = \rho\int_a^b x\,[f(x) - g(x)]\,dx$ and $M_x = \lim\limits_{n\to\infty} \sum_i \rho \cdot \frac{1}{2}\left[f(\overline{x}_i)^2 - g(\overline{x}_i)^2\right]\Delta x = \rho\int_a^b \frac{1}{2}\left[f(x)^2 - g(x)^2\right]dx$. Thus,

$$\overline{x} = \frac{M_y}{m} = \frac{M_y}{\rho A} = \frac{1}{A}\int_a^b x\,[f(x) - g(x)]\,dx \quad\text{and}\quad \overline{y} = \frac{M_x}{m} = \frac{M_x}{\rho A} = \frac{1}{A}\int_a^b \frac{1}{2}\left[f(x)^2 - g(x)^2\right]dx$$

8.4 Applications to Economics and Biology

1. $C(2000) = C(0) + \int_0^{2000} C'(x)\,dx = 1{,}500{,}000 + \int_0^{2000} \left(0.006x^2 - 1.5x + 8\right)dx$

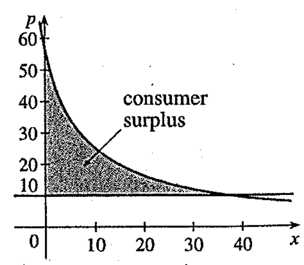
$= 1{,}500{,}000 + \left[0.002x^3 - 0.75x^2 + 8x\right]_0^{2000} = \$14{,}516{,}000$

3. $C'(x) = 74 + 1.1x - 0.002x^2 + 0.00004x^3$, so the increase in cost is

$$\begin{aligned} C(1600) - C(1200) &= \int_{1200}^{1600} \left(74 + 1.1x - 0.002x^2 + 0.00004x^3\right)dx \\ &= \left[74x + 0.55x^2 - \tfrac{0.002}{3}x^3 + 0.00001x^4\right]_{1200}^{1600} \\ &= 64{,}331{,}733.33 - 20{,}464{,}800 = \$43{,}866{,}933.33 \end{aligned}$$

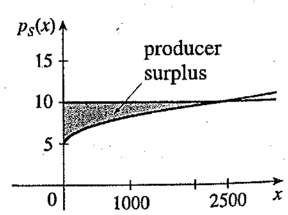
5. $p(x) = 10 = \dfrac{450}{x+8} \;\Rightarrow\; x + 8 = 45 \;\Rightarrow\; x = 37.$

$$\begin{aligned} \text{Consumer surplus} &= \int_0^{37} [p(x) - 10]\,dx = \int_0^{37} \left(\frac{450}{x+8} - 10\right)dx \\ &= [450\ln(x+8) - 10x]_0^{37} \\ &= 450\ln\left(\tfrac{45}{8}\right) - 370 \approx \$407.25 \end{aligned}$$



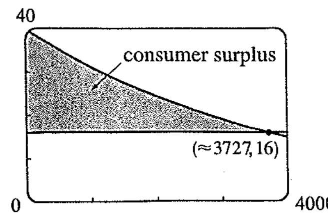
7. $P = p_S(x) = 10 = 5 + \frac{1}{10}\sqrt{x} \;\Rightarrow\; 50 = \sqrt{x} \;\Rightarrow\; x = 2500.$

$$\begin{aligned} \text{Producer surplus} &= \int_0^{2500} [P - p_S(x)]\,dx \\ &= \int_0^{2500} \left(10 - 5 - \tfrac{1}{10}\sqrt{x}\right)dx \\ &= \left[5x - \tfrac{1}{15}x^{3/2}\right]_0^{2500} \approx \$4166.67 \end{aligned}$$



9. $p(x) = \dfrac{800{,}000e^{-x/5000}}{x + 20{,}000} = 16 \;\Rightarrow\; x = x_1 \approx 3727.04.$

Consumer surplus $= \int_0^{x_1} [p(x) - 16]\,dx \approx \$37{,}753.01$



11. $f(8) - f(4) = \int_4^8 f'(t)\,dt = \int_4^8 \sqrt{t}\,dt = \left[\frac{2}{3}t^{3/2}\right]_4^8 = \frac{2}{3}\left(16\sqrt{2} - 8\right) \approx \$9.75$ million

13. $F = \dfrac{\pi P R^4}{8\eta\ell} = \dfrac{\pi\,(4000)\,(0.008)^4}{8\,(0.027)\,(2)} \approx 1.19 \times 10^{-4}\ \text{cm}^3/\text{s}$

15. $\int_0^{12} c(t)\,dt = \int_0^{12} \frac{1}{4}t\,(12 - t)\,dt = \left[\frac{3}{2}t^2 - \frac{1}{12}t^3\right]_0^{12} = (216 - 144) = 72\ \text{mg}\cdot\text{s/L}$. Therefore,
$F = A/72 = \frac{8}{72} = \frac{1}{9}\ \text{L/s} = \frac{60}{9}\ \text{L/min}.$

8.5 Probability

1. (a) $\int_{100}^{200} f(t)\,dt$ is the probability that a randomly chosen battery will have a lifetime of between 100 and 200 hours.

(b) $\int_{200}^{\infty} f(t)\,dt$ is the probability that a randomly chosen battery will have a lifetime of at least 200 hours.

3. (a) In general, we must satisfy the two conditions that are mentioned before Example 1 — namely, (1) $f(x) \geq 0$ for all x, and (2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$. Clearly, condition (1) is satisfied. For condition (2), we see that

$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{10} 0.1\,dx = \left[\frac{1}{10}x\right]_0^{10} = 1$. Thus, $f(x)$ is a probability density function.

(b) Since all the numbers between 0 and 10 are equally likely to be selected, we expect the mean to be halfway between the endpoints of the interval; that is, $x = 5$.

$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{10} x\,(0.1)\,dx = \left[\frac{1}{20}x^2\right]_0^{10} = \frac{100}{20} = 5$, as expected.

5. We need to find m so that $\int_m^{\infty} f(t)\,dt = \frac{1}{2} \;\Rightarrow\; \lim\limits_{x\to\infty} \int_m^x \frac{1}{5}e^{-t/5}dt = \frac{1}{2} \;\Rightarrow\; \lim\limits_{x\to\infty} \left[\frac{1}{5}(-5)\,e^{-t/5}\right]_m^x = \frac{1}{2} \;\Rightarrow$
$(-1)\left(0 - e^{-m/5}\right) = \frac{1}{2} \;\Rightarrow\; e^{-m/5} = \frac{1}{2} \;\Rightarrow\; -m/5 = \ln\frac{1}{2} \;\Rightarrow\; m = -5\ln\frac{1}{2} = 5\ln 2 \approx 3.47$ min.

7. We use an exponential density function with $\mu = 2.5$ min.

(a) $P(X > 4) = \int_4^{\infty} f(t)\,dt = \lim\limits_{x\to\infty} \int_4^x \frac{1}{2.5}e^{-t/2.5}\,dt = \lim\limits_{x\to\infty} \left[-e^{-t/2.5}\right]_4^x = 0 + e^{-4/2.5} \approx 0.202$

(b) $P(0 \leq X \leq 2) = \int_0^2 f(t)\,dt = \left[-e^{-t/2.5}\right]_0^2 = -e^{-2/2.5} + 1 \approx 0.551$

(c) We need to find a value a so that $P(X \geq a) = 0.02$, or, equivalently, $P(0 \leq X \leq a) = 0.98 \;\Leftrightarrow$
$\int_0^a f(t)\,dt = 0.98 \;\Leftrightarrow\; \left[-e^{-t/2.5}\right]_0^a = 0.98 \;\Leftrightarrow\; -e^{-a/2.5} + 1 = 0.98 \;\Leftrightarrow\; e^{-a/2.5} = 0.02 \;\Leftrightarrow$
$-a/2.5 = \ln 0.02 \;\Leftrightarrow\; a = -2.5\ln\frac{1}{50} = 2.5\ln 50 \approx 9.78$ min ≈ 10 min. The ad should say that if you aren't served within 10 minutes, you get a free hamburger.